

Commutative Algebra Exercises Solutions

Commutative Algebra Exercises Solutions Commutative Algebra Exercises Solutions This document provides solutions to a selection of exercises in commutative algebra. The exercises are chosen to cover a wide range of topics from basic definitions to more advanced concepts. The solutions are detailed and aim to provide a thorough understanding of the underlying concepts. This document is organized into sections, each focusing on a specific topic in commutative algebra. Each section contains a brief overview of the topic followed by a selection of solved exercises.

Sections

- 1 Rings and Ideals** Overview: This section introduces the basic definitions of rings and ideals along with fundamental properties. Exercises: Prove that the set of even integers forms an ideal in the ring of integers. Determine whether the ideal generated by $x^2 + 1$ is a prime ideal in the polynomial ring $\mathbb{Z}[x]$. Show that the intersection of two ideals is an ideal. Find all the ideals of the ring $\mathbb{Z}/6\mathbb{Z}$.
- 2 Modules and Homomorphisms** Overview: This section introduces the concept of modules, a generalization of vector spaces over fields. Homomorphisms between modules are also discussed. Exercises: Determine whether the set of all polynomials with even coefficients forms a submodule of the module of all polynomials over the ring of integers. Show that the set of all matrices of the form $\begin{pmatrix} a & b & 0 \\ 0 & c & 0 \end{pmatrix}$ where a, b, c are real numbers forms a module over the ring of real numbers. Prove that the homomorphism theorem holds for modules.
- 3 Localization and Quotient Rings** Overview: This section explores the concepts of localization and quotient rings, which provide tools for constructing new rings from existing ones. Exercises: Find the localization of the ring of integers at the prime ideal 2 . Show that the quotient ring $\mathbb{Z}[x]/(x^2 + 1)$ is isomorphic to $\mathbb{Z} \times \mathbb{Z}$. Prove that the localization of a ring at a prime ideal is a local ring. Determine the units in the localization of the ring of integers at the prime ideal 5 .
- 4 Noetherian Rings and Modules** Overview: This section introduces the concept of Noetherian rings and modules, which are crucial for studying ideals and submodules. Exercises: Show that the ring of integers is a Noetherian ring. Prove that any finitely generated module over a Noetherian ring is Noetherian. Demonstrate that the Hilbert Basis Theorem holds for polynomial rings over Noetherian rings. Find a ring that is not Noetherian.
- 5 Integral Domains and Field Extensions** Overview: This section delves into integral domains and field extensions, which are central to algebraic number theory and algebraic geometry. Exercises: Show that the ring of Gaussian integers is an integral domain. Determine the degree of the field extension $\mathbb{Q}(\sqrt{2})/\mathbb{Q}$. Prove that any finite integral domain is a field. Find the minimal polynomial of $\sqrt[3]{2}$ over the field of rational numbers.
- 6 Dimension Theory** Overview: This section introduces the notion of dimension for rings and modules, which measures their complexity in terms of chains of ideals or submodules. Exercises: Calculate the Krull dimension of the ring of integers. Determine the dimension of the polynomial ring $\mathbb{C}[x, y]$. Prove that the dimension of a Noetherian ring is finite. Show that the dimension of a module is less than or equal to the dimension of the ring over 3 which it is defined.
- 7 Primary Decomposition and Associated Primes** Overview: This section explores primary decomposition, a powerful tool for understanding ideals.

in commutative rings and the concept of associated primes Exercises Find a primary decomposition of the ideal $x^2 xy$ in the polynomial ring $\mathbb{C}[x, y]$ Determine the associated primes of the ideal $x^2 xy$ in $\mathbb{C}[x, y]$ Prove that the radical of a primary ideal is a prime ideal Show that any ideal in a Noetherian ring has a primary decomposition 8 Unique Factorization Domains and Dedekind Domains Overview This section discusses the concept of unique factorization domains UFDs which generalize the familiar factorization of integers into primes and Dedekind domains a class of rings with important properties in algebraic number theory Exercises Prove that the polynomial ring $\mathbb{C}[x]$ is a UFD Show that the ring of integers of a quadratic field is a Dedekind domain Find the prime factorization of the ideal 2 in the ring of integers of the quadratic field $\mathbb{Q}(\sqrt{5})$ Determine whether the ring $\mathbb{Z}[\sqrt{3}]$ is a UFD 9 Modules over Principal Ideal Domains Overview This section focuses on modules over principal ideal domains PIDs which have a rich structure and play a significant role in linear algebra Exercises Classify all finitely generated modules over the ring of integers Prove that any finitely generated module over a PID is a direct sum of cyclic modules Determine the invariant factors of the module $\mathbb{Z}^6 \oplus \mathbb{Z}^{12}$ Find the Smith normal form of the matrix $\begin{pmatrix} 2 & 4 & 6 & 8 \end{pmatrix}$ over the ring of integers 10 Applications of Commutative Algebra Overview This section explores various applications of commutative algebra in other areas of mathematics including algebraic geometry number theory and coding theory 4 Exercises Use Hilbert's Nullstellensatz to show that the ideal x^2y^2 in $\mathbb{C}[x, y]$ defines the unit circle in the complex plane Apply the Chinese Remainder Theorem to solve the system of congruences $x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$ Use the theory of Gröbner bases to find the solution to the system of equations $x^2 + y^2 = 1$, $x + y = 1$ Note The selection of exercises and their level of difficulty will vary depending on the target audience and the scope of the course This document provides a general framework for structuring a collection of commutative algebra exercise solutions

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